members looked at each other, stunned. 'And by the way', added Bowles, 'you can delete paragraph 1 of the constitution'.

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Mixed strategies can't evade Pascal's Wager

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1. The goal of this article is to defend Pascal's Wager from a particular way of evading it, the mixed strategy approach. After explaining Pascal's Wager and the mixed strategy way of evading it, I will show that while there's nothing technically wrong with the mixed strategy approach, rationality requires it to be applied in such a way that Pascal's Wager doesn't lose any force.

Pascal, in giving his Wager, attempts to give those atheists and agnostics who initially assign a non-zero probability to the hypothesis that God exists pragmatic reason to believe in God. He argues that an agent who follows the strategy of believing in God maximizes the agent's expected utility. The expected utility an agent has for believing in God is the sum of: (1) the agent's subjective probability that God exists, multiplied by the utility of believing in God, under the supposition that God exists, and (2) the agent's subjective probability that God does not exist, multiplied by the utility of believing in God, under the supposition that God does not exist. This expected utility is infinite, since the utility of believing in God, under the supposition that God exists, is infinite, and the utility of believing in God, under the supposition that God doesn't exist, is some finite quantity. Moreover, Pascal maintains that the expected utility of not believing in God is finite, since only believers get the infinite rewards of heaven. Pascal thus maintains that a pragmatically rational agent, who desires to maximize expected utility, will follow the strategy of believing in God.

Duff (1986), with elaborations by Hájek (2003), has ingeniously pointed out that one can get the same infinite expected utility from other strategies besides the strategy of believing in God. Specifically, Hájek proposes a mixed strategy, where (for example) one decides to roll a fair n-sided die, and to believe in God if the die lands on side #1, and to not believe in God if the die lands on any other side. The expected utility of following this mixed strategy is $1/n$ times the expected utility of believing in God, plus $(n-1)/n$ times the

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expected utility of not believing in God. As long as n is finite, this expected utility will still be infinite. It follows that an agent who follows the mixed strategy has the same expected utility as an agent who follows the strategy of believing in God. Thus, it seems that atheists and agnostics who aren't in the mood to believe in God can follow a mixed strategy with large n, and get the same expected utility as following Pascal's preferred strategy of believing in God, while probably avoiding the trouble of actually believing in God. As Hájek (2003: 31) writes,

> Pascal has ignored all these mixed strategies – probabilistic mixtures of the 'pure actions' of wagering for and wagering against God . . . Nothing in his argument favours wagering for God over all of these alternative strategies.

Duff (1986: 109) suggests that this line of reasoning provides 'a *reductio ad absurdum* of the Wager'.

2. The problem with the mixed strategies objection is that an agnostic or atheist has more than one opportunity to choose to follow a mixed strategy. Suppose that you are faced with Pascal's Wager, and you decide to follow the mixed strategy of rolling a die, and believing in God if #1 is rolled, and otherwise continuing with your atheism or agnosticism. Before the die is rolled, this choice of yours has infinite expected utility. But after the die is rolled, and #1 does not come up, the expected utility of following that course of action changes – it is no longer infinite. This is because new information is available that leads you to make a new calculation for the expected utility of following that course of action. You are back to where you started – as an atheist or agnostic, without the prospect of getting that eternal reward for believing in God.

But given that you are back to where you started, and it was rational for you to follow the mixed strategy the first time, then it is rational for you to follow the mixed strategy again. And if the die does not land on #1 again, you will be back to where you started again, so it is rational for you to follow the mixed strategy yet again. This process will continue, until the die does land on #1, and you end up believing in God.

If you follow this mixed strategy with a die with a large number of sides, this process could take a while. There are three things to note in this context. First, the number of sides has to be finite, for there to be an infinite expected utility of following this mixed strategy. Infinite utility is being multiplied by $1/n$, and only as long as n is finite is this result still infinite.

Second, since n has to be finite, you should expect with probability 1 that the side #1 will be rolled after a finite number of attempts. This finite number of attempts can take place arbitrarily quickly, since in any arbitrarily small interval of time there are an infinite number of instants – continuum-many, in fact.

Third, it may be the case that in practice you would not be able to engage in die-rolls that quickly. But you can see where the process will end up, if you were to keep engaging in it. Thus, it seems pragmatically rational not to keep sitting there and rolling the die, but instead to embrace the result that one fully expects to eventually get, and hence choose to believe in God.

3. Hájek does make one claim, in an endnote, that can be taken to implicitly address the argument I've given above. Suppose that you follow the mixed strategy by rolling the *n*-sided die, and side #1 does not come up, so you do not decide to believe in God. Hájek (2003: n. 11) says that in this situation,

> your expectation does not change. By Pascal's lights, you still enjoy infinite expectation whatever you do next.

I maintain that Hájek is mistaken. Your choice to follow the mixed strategy *had* a certain expected utility. It doesn't follow that, going forward, that choice now has that expected utility.

Consider a simple finite example. Suppose that I give you a ticket that has a 10% chance of winning $500 in a draw that will take place next Monday. Currently, your expected utility of owning this ticket is $50. But Monday rolls around, and you lose the drawing. It's not the case that there is still an expected utility of $50 associated with owning that ticket. Because of the outcome of a chance event, your expected utility changes. Now, the expected utility of owning that ticket is $0.

The same sort of reasoning holds when one is following a mixed strategy. At the time one chooses to follow the mixed strategy, the expected utility of following that course of action is infinite, but the expected utility changes, based on the outcome of a chancy event. In calculating an agent's expected utility for following a certain course of action, one has to take into account the probabilities the agent assigns to each possible outcome, together with the utilities the agent associates with each outcome. When the agent learns new evidence, the agent should update on that evidence, in good Bayesian fashion, and as a result the agent's probabilities for the possible outcomes can change. This new evidence should be taken into account when the agent is evaluating the current expected utility for a course of action the agent has chosen in the past to follow.

4. I'll end with three caveats. First, my discussion does not apply to all versions of Pascal's Wager. For example, Hájek (2003) presents various reformulations of Pascal's Wager, reformulations which have the virtue of not falling prey to the mixed strategies evasion. Hájek presents these reformulations in large part because he sees the mixed strategy approach as damning to the original formulation of Pascal's Wager. The distinctive thing I have shown is that the reformulations aren't needed, at least not to deal with the mixed strategy approach – the mixed strategy approach does not pose a significant problem for Pascal's original argument.

Second, note that I have not absolved Pascal's Wager of all objections that could be raised against it. For example, here is one objection that I have not seen in the literature before, but is worth noting. Consider the option of following a *delayed* strategy. On this strategy, you decide now to stay an atheist or agnostic for the next, say, 50 years, and *then* choose to believe in God. The expected utility of following this course of action is infinite, even though there is a significant chance that you will be dead 50 years from now, and hence will never get the opportunity to believe in God. As long as you assign a non-zero probability to the hypothesis that you will be alive 50 years from now, your expected utility of deciding now to believe in God 50 years from now is infinite. My objection to the mixed strategy response to Pascal's Wager does not carry over to the delayed strategy response, since the expected utility of following that course of action doesn't change for the next 50 years. (And you could pick the number of years to be as high as you like, as long as you assign a non-zero probability to the hypothesis that you will live that long.)

Here is the third and final caveat. I have not dealt with issues that arise under the plausible supposition that you cannot just choose to believe in God, and thus instead the strategy you would have to follow is the strategy of *trying* to believe in God. Now, it's true that as long as the strategy of trying to believe has a non-zero probability of succeeding, then the expected utility of following that strategy is infinite. But (as Duff has pointed out) there is a problem that arises in this context. The expected utility of *not* trying to believe in God is also infinite, since there's (arguably) a non-zero probability that you will end up believing in God, even if you are not trying. My strategy for responding to the attempt to evade Pascal's Wager via mixed strategies does not solve this other problem for Pascal's Wager. For agents who have a non-zero probability of believing in God even if they do not try to believe, the expected utility of living their everyday atheistic or agnostic life is arguably infinite. For such agents, nothing I have said in this article provides them with any incentive to try to believe. But for agents for whom belief and unbelief in God is a matter of the will, the mixed strategy approach won't prevent them from being obligated by Pascal's Wager to choose to believe in God.

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References

Duff, A. 1986. Pascal's Wager and infinite utilities. *Analysis* 46: 107–9.

Hájek, A. 2003. Waging war on Pascal's Wager. *Philosophical Review* 112: 27–56.